# Technical Comments

AIAA 82-4089

## Comment on "Analytical Prediction of Vortex Lift"

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SEVERAL comments are warranted concerning the paper by J.W. Purvis entitled "Analytical Prediction of Vortex Lift," which appears in the April 1981 issue of the *Journal of Aircraft.* The first comment is relative to the use of singular loadings in the design of mean camber shapes for wings experiencing leading-edge-vortex separated flow. By comparison, Ref. 1 outlines a low-drag, separated-flow design procedure which uses a more appropriate attached-flow initial loading that is nonsingular. There the final solution for the mean camber surface is obtained by *iterating* to find the shape-yielding minimum separated-flow drag at the required lift coefficient.

The second comment is that the section-induced drag, see Eq. (12), does not have an elliptical distribution *over the wing* as proposed, but only in the Trefftz plane. This is well known (e.g., Ref. 2), and means that for most wings of interest the section-induced drag has to be determined in the near field. The consequence of this wrong assumption leads to an error in Eq. (26) and, hence, in the magnitude of the leading-edge suction force and vortex flow aerodynamics for wings with varying leading-edge sweep.

The third comment deals with the missing multiplication by 2 in Eqs. (27), (30), (33), and (34), the missing division by 2 in Eqs. (28) and (37), and the typographical error in Eq. (32). Equation (32) should read

$$E_1 = \frac{1}{\pi/2 - 2/3} \cong 1.106036$$

The missing multiplication by 2 in Eqs. (27), (30), (33), and (34) is just canceled by the missing division by 2 in the definition for $H(\eta)$. However, this is not the case in Eq. (37). From Ref. 3, Eq. (37) should be of the form

$$s(x) = \rho\pi(b/2)\lim_{\eta\to 1}(1-\eta)v^2 \equiv \rho\pi(b/2)\lim_{\eta\to 1}(1-\eta)(\delta^2/4)$$

This error in Eq. (37) will have the effect of making the side-edge suction force and the associated vortex-lift/moment too large by a factor of 2.

As a consequence of the foregoing comments, the resulting agreement or disagreement of this method with data for wings of varying leading-edge sweep and for wings with side edges is unconvincing.

### References

[1] Lamar, J.E., Schemensky, R.T., and Reddy, C.S., "Development of a Vortex-Lift Design Procedure and Application to a Slender Maneuver Wing Configuration," *Journal of Aircraft,* Vol. 18, April 1981, pp. 259-266.

[2] Garner, H.C., "Some Remarks on Vortex Drag and Its Spanwise Distribution in Incompressible Flow," British NPL Aero Note 1048, 1966.

[3] von Kármán, T. and Burgers, J.M., "General Aerodynamic Theory-Perfect Fluids," *Aerodynamic Theory,* Vol. II, Division E, Sec. 10, Julius Springer, Berlin, 1935, pp. 48-53.

Received Aug. 31, 1981. 

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AIAA 82-4090

## Reply by the Author to J. E. Lamar

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THE author would like to thank Mr. Lamar for his comments concerning "Analytical Prediction of Vortex Lift."[1] There are several typographical errors in the paper which may have contributed to his confusion. With respect to Eq. (24), Eq. (25) obviously should read

$$cc_s\cos\Lambda = \frac{b}{2}E_0\int_0^\eta (cc_l\sin\alpha - cc_d\cos\alpha)\,d\eta_0$$

In Eq. (26), $w_i$ should be $w_0$, as shown by direct substitution from Eqs. (2) and (12). Similarly, the required substitutions between Eqs. (26) and (27) show that Eq. (28) should obviously read

$$H(\eta) = 2\int_0^\eta \sqrt{1-\eta_0^2}\,d\eta_0 = \eta\sqrt{1-\eta^2} + \arcsin(\eta)$$

Finally, as noted by Mr. Lamar, Eq. (32) should read

$$E_1 = \frac{1}{\pi/2 - 2/3} \cong 1.106036$$

The author does not understand the comparison mentioned in Mr. Lamar's first comment, since there is no design procedure presented in Ref. 1. The nonsingular attached-flow initial loading statement is also unclear, since the leading edges of thin wings, whether cambered or not, always exhibit theoretical pressure loading singularities in linearized potential flow. Vortex lattice schemes, of course, will only approach this behavior in the limit as the number of chordwise vortices becomes large.

Even after the extensive review and rebuttal process, Mr. Lamar still seems to be confused by the section of the paper dealing with the spanwise distribution of vortex lift. Garner[2] computes section drag from the small angle formula

$$cc_{d_i} = cc_l\alpha - \tfrac{1}{8}\pi\sec\Lambda\lim_{x\to x_{\rm LE}}(\Delta C_p)^2(x - x_{\rm LE})$$

where the second term on the right-hand side is the local leading-edge suction (acting in the plane of the wing). But the fundamental assumption of the suction analogy is that the leading-edge suction force acts *normal* to the chord, whereupon the section drag tends to that given by the usual Trefftz plane calculation. Under the assumption of an elliptic lift distribution, the Trefftz plane section drag formula is then Eq. (12). The accuracy of Eq. (26) [and (27)] is shown in Fig. 1, where the distribution given by Eq. (27) is compared with numerical results by Snyder and Lamar[3] for a low aspect ratio delta wing. The distributions are remarkably close, considering the number of "errors" and wrong "assumptions" in the present analysis.

Received Oct. 16, 1981. 

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Table 1 $C_{L_{VSE}}$ results using Eq. (52) compared with Lamar [4]

| | Rectangular wings | | | | | Cropped deltas, $\Lambda = 63$ deg | | | |
|---|---|---|---|---|---|---|---|---|---|
| $AR$ | 0.2 | 0.3 | 0.4 | 1.0 | 3.0 | 1.64 | 1.33 | 1.08 | 0.86 |
| $\alpha$,[a] deg | 30 | 30 | 30 | 30 | 12 | 26 | 26 | 26 | 28 |
| $K_p$[b] | 0.3147 | 0.4667 | 0.6200 | 1.4667 | 3.1419 | 2.1743 | 1.8778 | 1.5870 | 1.3060 |
| $c_t$,[b] cm | 127.0 | 101.6 | 76.2 | 50.80 | 25.40 | 1.709 | 3.50 | 5.43 | 7.55 |
| $b$,[b] cm | 25.4 | 30.48 | 30.48 | 50.80 | 76.20 | 22.89 | 20.32 | 17.80 | 15.27 |
| $C_{L_{VSE}}$[b] | 0.6070 | 0.5900 | 0.5675 | 0.4600 | 0.0500 | 0.0700 | 0.1300 | 0.195 | 0.288 |
| $C_{L_{VSE}}$ [Eq. (52)] | 0.6294 | 0.6153 | 0.6109 | 0.547 | 0.0695 | 0.0699 | 0.1448 | 0.2180 | 0.3136 |
| Difference, % | 1.8 | 2.1 | 3.7 | 8.6 | 16.3 | −0.1 | 5.4 | 5.6 | 4.3 |
| $C_{L_{VSE}}$[b]$/C_{L_{TOTAL}}$,[b] % | 80.4 | 72.4 | 65.2 | 39.0 | 6.5 | 5.3 | 10.8 | 18.0 | 27.2 |

[a] Maximum angle of attack given in Ref. 4. [b] Values taken from Ref. 4.

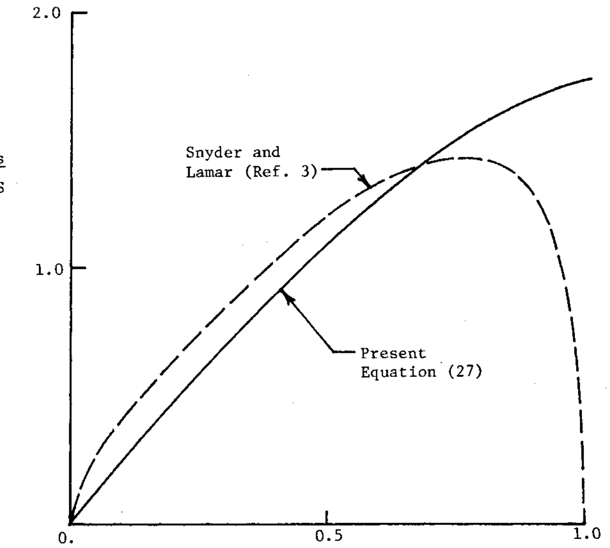


Fig. 1 Suction distribution for $AR = 1.147$ delta wing.

With the exception of Eq. (37), the phantom "2"'s in Mr. Lamar's third comment all result from the previously mentioned typographical errors in Eqs. (28) and (32). Concerning Eq. (37), Mr. Lamar is quite correct that the usual formula for computing side-edge suction is

$$s(x) = \pi\rho(b/2)\lim_{\eta \to 1}(1-\eta)(\delta^2/4)$$

when the actual form of $\delta(\eta)$ or $\Gamma(\eta)$ is known. Using the pressure loading form assumed in Ref. 1, however, the side-edge suction distribution is more closely approximated by Eq. (37), which differs from the usual form by a factor of 2.

The discrepancy between the two forms is due to the fact that $\delta$ is a gradient, $\delta(\eta) = (2/b)(\partial\Gamma(\eta)/\partial\eta)$, where the form of $\Gamma(\eta)$ is

$$\Gamma(\eta) = f(\eta)\sqrt{1-\eta^2}$$

and $f(\eta)$ is a function which may be expressed as

$$f(\eta) = a_0 + a_1\eta + a_2\eta^2 + \ldots$$

The assumption of Ref. 1 that $f(\eta) = a_0$ is sufficient for accurate calculation of such terms as total lift or lift distribution, since the other terms in $f(\eta)$ affect the magnitude of $\Gamma(\eta)$ very little. The gradient of $\Gamma(\eta)$ at the wing tips, however, is significantly influenced by the other terms in $f(\eta)$. For example, when $f(\eta)$ is given by the above series, then

$$\lim_{\eta \to 1}(1-\eta)\left(\frac{\partial\Gamma}{\partial\eta}\right)^2 = \frac{a_0^2}{2}[1 + a_1/a_0 + a_2/a_0 + \ldots]^2$$

During development of the theory presented in Ref. 1, it was found that for a large variety of wings the term in brackets is approximately 2. As an example, consider the two-term function given by Garner[2] for the aspect ratio 4, hyperbolic sweep wing. Using Garner's notation,

$$f(\eta) = (A_1 - A_3) + 4A_3\eta^2$$

which gives

$$a_0 = A_1 - A_3 \qquad \frac{a_2}{a_0} = \frac{4(A_3/A_1)}{1-(A_3/A_1)}$$

and thus

$$[1 + a_2/a_0]^2 = 2.26\ldots$$

The sentence immediately preceding Eq. (37) was meant to imply that Eq. (37) was not the usual form, but rather a form that should be used specifically with the assumed pressure function of Ref. 1.

If Mr. Lamar remains unconvinced about the accuracy of the present method, he should examine Table 1, wherein $C_{L_{VSE}}$ is computed for a number of wings using Eq. (52). The results are compared to Mr. Lamar's own calculations of $C_{L_{VSE}}$ from Ref. 4. To minimize the possibility of error, the geometric parameters and the potential constant used in Eq. (52) were taken directly from Ref. 4. Note that the greatest disagreement between the two methods occurs for high $AR$ wings, where $C_{L_{VSE}}$ is a small percentage of the total lift. In general, the results using Eq. (52), are comparable to those presented in Figs. 4 and 5 of Ref. 1. Similar results can be obtained for wings with varying leading-edge sweep by using Eq. (34) and existing data such as that of Ref. 5.

## References

[1] Purvis, J.W., "Analytical Prediction of Vortex Lift," *Journal of Aircraft*, Vol. 18, April 1981, pp. 225-230.

[2] Garner, H.C., "Some Remarks on Vortex Drag and its Spanwise Distribution in Incompressible Flow," *Aeronautical Journal*, Vol. 72, July 1968, pp. 623-625.

[3] Snyder, M.H. and Lamar, J.E., "Application of the Leading-Edge-Suction Analogy to Prediction of Longitudinal Load Distribution and Pitching Moments for Sharp-Edged Delta Wings," NASA TN D-6994, Oct. 1972.

[4] Lamar, J.E., "Extension of Leading-Edge-Suction Analogy to Wings with Separated Flow Around the Side Edges at Subsonic Speeds," NASA TR R-423, Oct. 1974.

[5] Polhamus, E.C., "Prediction of Vortex-Lift Characteristics by a Leading Edge Suction Analogy," *Journal of Aircraft*, Vol. 8, April 1971, pp. 193-200.